<u>**Unit – III (Methods to find the basic feasible solution (bfs) for a balanced TP)**</u>

There are three basic methods:

1. Northwest Corner Method
2. Minimum Cost Method
3. Vogel's Method

1. Northwest Corner Method: To find the bfs by the NWC method:

Begin in the upper left (northwest) corner of the transportation tableau and set x_{11} as large as possible (here the limitations for setting x_{11} to a larger number, will be the demand of demand point 1 and the supply of supply point 1. Your x_{11} value can not be greater than minimum of this 2 values).
According to the explanations given above we can set x_{11}=3 (meaning demand of demand point 1 is satisfied by supply point 1).

				5
				6
				2
3	5	2	3	

3				2
				6
				2
X	5	2	3	

After we check the east and south cells, we saw that we can go east (meaning supply point 1 still has capacity to fulfill some demand).

3	2			X
				6
				2
X	3	2	3	

3	2			X
	3			3
				2
X	X	2	3	

After applying the same procedure, we saw that we can go south this time (meaning demand point 2 needs more supply by supply point 2).

3	2			X
	3	2		1
				2
X	X	X	3	

3	2			X
	3	2	1	X
				2
X	X	X	2	

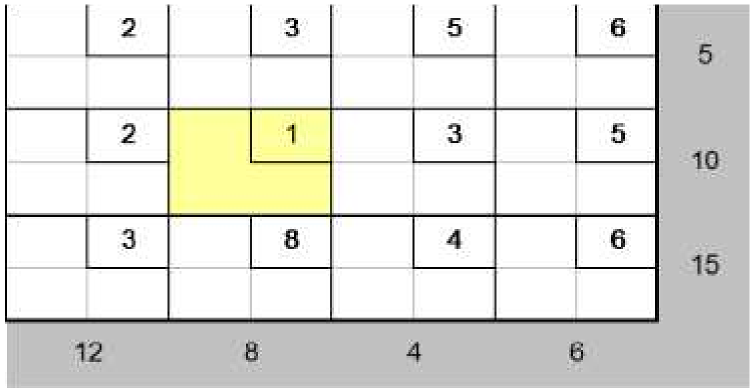
Finally, we will have the following bfs, which is:
$x_{11}=3$, $x_{12}=2$, $x_{22}=3$, $x_{23}=2$, $x_{24}=1$, $x_{34}=2$

3	2			X
	3	2	1	X
			2	X
X	X	X	X	

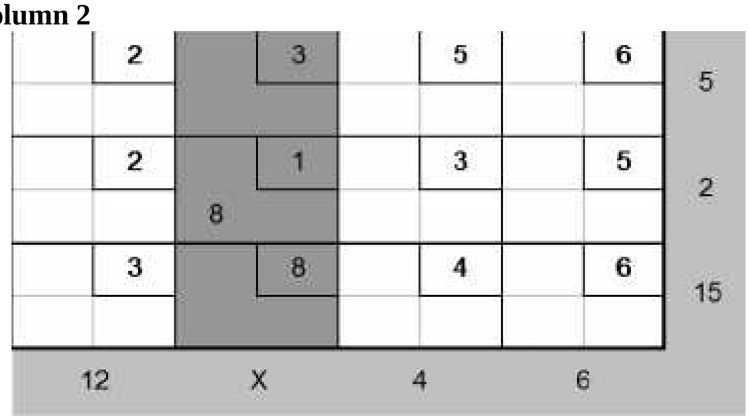
2. Minimum Cost Method

The Northwest Corner Method dos not utilize shipping costs. It can yield an initial bfs easily but the total shipping cost may be very high. The minimum cost method uses shipping costs in order come up with a bfs that has a lower cost. To begin the minimum cost method, first we find the decision variable with the smallest shipping cost (X_{ij}). Then assign X_{ij} its largest possible value, which is the minimum of s_i and d_j After that, as in the Northwest Corner Method we should cross out row i and column j and reduce the supply or demand of the noncrossed-out row or column by the value of Xij. Then we will choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and we will repeat the procedure.

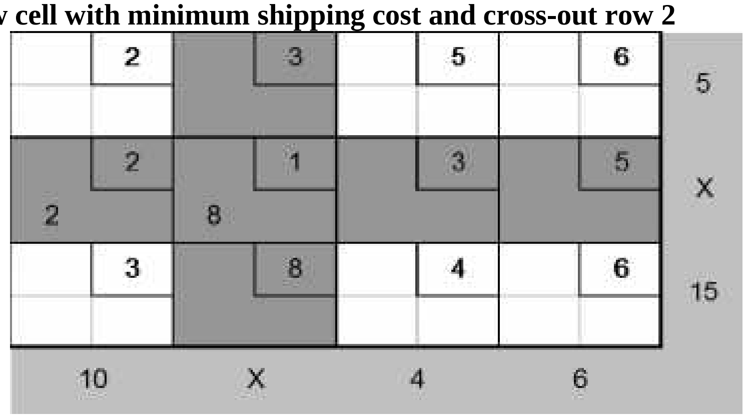
An example for Minimum Cost Method
Step 1: Select the cell with minimum cost.



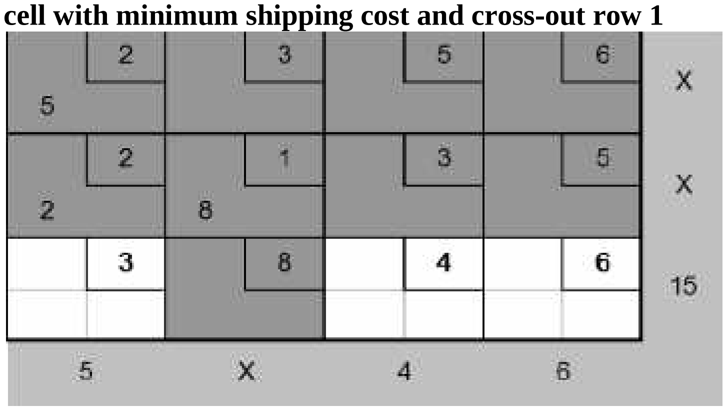
Step 2: Cross-out column 2



Step 3: Find the new cell with minimum shipping cost and cross-out row 2



Step 4: Find the new cell with minimum shipping cost and cross-out row 1



Step 5: Find the new cell with minimum shipping cost and cross-out column 1

2 / 5	3	5	6	X
2 / 2	1 / 8	3	5	X
3 / 5	8	4	6	10
X	X	4	6	

Step 6: Find the new cell with minimum shipping cost and cross-out column 3

2 / 5	3	5	6	X
2 / 2	1 / 8	3	5	X
3 / 5	8	4 / 4	6	6
X	X	X	6	

Step 7: Finally assign 6 to last cell. The bfs is found as: $X_{11}=5$, $X_{21}=2$, $X_{22}=8$, $X_{31}=5$, $X_{33}=4$ and $X_{34}=6$

2 / 5	3	5	6	X
2 / 2	1 / 8	3	5	X
3 / 5	8	4 / 4	6 / 6	X
X	X	X	X	

3. Vogel's Method

Begin with computing each row and column a penalty. The penalty will be equal to the difference between the two smallest shipping costs in the row or column. Identify the row or column with the largest penalty. Find the first basic variable which has the smallest shipping cost in that row or column. Then assign the highest possible value to that variable, and cross-out the row or column as in the previous methods. Compute new penalties and use the same procedure.

An example for Vogel's Method

Step 1: Compute the penalties.

				Supply	Row Penalty
	6	7	8	10	7-6=1
	15	80	78	15	78-15=63
Demand	15	5	5		
Column Penalty	15-6=9	80-7=73	78-8=70		

Step 2: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 (5)	8	5	8-6=2
	15	80	78	15	78-15=63
Demand	15	X	5		
Column Penalty	15-6=9	–	78-8=70		

Step 3: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 (5)	8 (5)	0	–
	15	80	78	15	–
Demand	15	X	X		
Column Penalty	15-6=9	–	–		

Step 4: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6 (0)	7 (5)	8 (5)	X	–
	15	80	78	15	–
Demand	15	X	X		
Column Penalty	–	–	–		

Step 5: Finally the bfs is found as X_{11}=0, X_{12}=5, X_{13}=5, and X_{21}=15

				Supply	Row Penalty
	0 (6)	5 (7)	5 (8)	X	–
	15 (15)	(80)	(78)	X	–
Demand	X	X	X		
Column Penalty	–	–	–		

EXAMPLE

Determine an initial basic feasible solution to the following transportation problem using the North-West corner rule:

	D_1	D_2	D_3	D_4	**Availability**
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requirement	6	10	15	4	

Solution to the above problem is:

6	8			
6	4	1	5	
		2	14	
8	9	2	7	
			1	4
4	3	6	2	

Now all requirements have been satisfied and hence an initial basic feasible solution to the transportation problem has been obtained. Since the allocated cells do not form a loop, the feasible solution is non-degenerate. Total transportation cost with this allocation is:

Z = 6*6 + 4*8 + 2*9 +14*2 + 1*6 +4* = 128.

Least Cost Method

The allocation according to this method is very useful as it takes into consideration the lowest cost and therefore, **reduce the computation** as well as the amount of time necessary to arrive at the optimal solution.

Step 1

(a) Select the cell with the lowest transportation cost among all the rows or columns of the transportation table.

(b) If the minimum cost is not unique, then select arbitrarily any cell with this minimum cost.

Step 2

Allocate as many units as possible to the cell determined in Step 1 and eliminate that row (column) in which either supply is exhausted or demand is satisfied.

Repeat Steps 1 and 2 for the reduced table until the entire supply at different plants is exhausted to satisfied the demand at different warehouses.

	Ware House			Supply
	W_1	W_2	W_3	S_i
P_1	[7]	[6] (20)	[9]	20 0
P_2	[5] (4)	[7] (5)	[3] (19)	~~28~~ ~~9~~ ~~5~~ 0
P_3	[4] (17)	[5]	[8]	~~17~~ 0
Demand D_j	~~21~~ ~~4~~ 0	~~25~~ ~~5~~ 0	~~19~~ 0	65

Total cost = 20 x 6 + x 5 + 5 x 7 + 19 x 3 + 17 x 4 = Rs. 300

This total cost is less than the total cost obtained by NWCM.

VOGEL'S APPROXIMATION METHOD (VAM).

Step 1. For each row of the transportation table, identify the smallest and the next to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 2. Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to i^{th} row and the minimum cost be C_{ij} . Allocate a maximum feasible amount $x_{ij} = min (a_i , b_j)$ in the $(i, j)^{th}$ cell, and cross off the i^{th} row or j^{th} column.

Step 3. Re compute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

Remark. VAM determines an initial basic feasible solution, which is very close to the optimum solution.

PROBLEM

Obtain an initial basic feasible solution to the following transportation problem using Vogels approximation method.

	I	II	III	IV	
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	-1	4	2	19
	21	25	17	17	

Find the initial basic feasible solution by using North-West Corner Rule, least cost & Vogel's method.

1.

W→ F ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

2.

	D_1	D_2	D_3	D_4	Supply
O_1	1	5	3	3	34
O_2	3	3	1	2	15
O_3	0	2	2	3	12
O_4	2	7	2	4	19
Demand	21	25	17	17	80

3.

From	To					Supply
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	1	4	8	12	9
Demand	3	3	4	5	6	

4.

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	80	70	20	18
Requirement	5	8	7	14	

5.

	A	B	C	Availability
I	50	30	220	1
II	90	45	170	4
III	250	200	50	4
Requirement	4	2	3	

6.

		Stores				Availability
		I	II	III	IV	
	A	21	16	15	13	11
Warehouse	B	17	18	14	23	13
	C	32	27	18	41	19
Requirement		6	10	12	15	

7 Determine an initial basic feasible solution to the following transportation problem using north-west corner rule.

		I	II	III	IV	Supply
	A	13	11	15	20	2000
From	B	17	14	12	13	6000
	C	18	18	15	12	7000
	Demand	3000	3000	4000	5000	

[Ans. $x_{11} = 2$, $x_{21} = 1$, $x_{22} = 3$, $x_{23} = 2$, $x_{34} = 5$]

8 Obtain an initial basic feasible solution to the following transportation problem using matrix minima method.

		D1	D2	D3	D4	Capacity
	O1	1	2	3	4	6
From	O2	4	3	2	0	8
	O3	0	2	2	1	10
	Demand	4	6	8	6	

[Ans. $x_{12} = 6$, $x_{23} = 2$, $x_{24} = 6$, $x_{31} = 4$, $x_{32} = 0$, $x_{33} = 6$]

9 Determine the minimum cost to the following transportation problem using Vogel's method.

		D1	D2	D3	D4	D5	Capacity
	O1	2	11	10	3	7	4
From	O2	1	4	7	2	1	8
	O3	3	9	4	8	12	9
	Demand	3	3	4	5	6	21

[Ans. Min cost = Rs 68]

10 Determine the minimum cost to the following transportation problem using matrix minima method and vogel's method

		D1	D2	D3	D4	Capacity
	O1	1	2	1	4	30
From	O2	3	3	2	1	50
	O3	4	2	5	9	20
	Demand	20	40	30	10	

[Ans. Min cost = Rs 180]

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To be continued -------