

2010

Time : 3 hours

Full Marks : 80

*Candidates are required to give their answers in
their own words as far as practicable.*

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Objective Type Questions)

Answer **all** questions.

1. Choose the correct answer of the following : 20
 - (a) Let $A = \{1, 2, 3, \dots, 8, 9\}$ and $B = \{3, 5, 7, 9\}$,
Which of the following statements is correct ?
 - (i) $A \cap B = \{2, 4, 6\}$
 - (ii) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iii) $A \cap B = \{1, 2, 4, 6, 8\}$

(iv) $A \cap B = \{2, 4, 6, 8\}$

(b) Let $A \subset B$, $B \subset C$ and $D \cup A = C$. Which statement is always false ?

(i) $B \subset D$

(ii) $A \subset C$

(iii) $A = B$

(iv) $B \cap D = \phi$ and $B = A$

(c) A survey shows that 63% of the people watch a News Channel whereas 76% watch another channel. If $x\%$ of the people watch both channel, then :

(i) $x = 35$

(ii) $x = 63$

(iii) $39 \leq x \leq 63$

(iv) $x = 39$

(d) If A , B and C are matrices with orders 3×3 , 2×3 and 4×2 respectively, how many of the following matrix calculations are possible ?

$4B$, $A + B$, $3B^T + C$, AB , $B^T A$, $(CB)^T$, CBA

(i) 0

(ii) 1

(iii) 2

(iv) 3

(e) Find the determinant of the matrix :

$$\begin{bmatrix} 5 & -2 & 3 \\ 4 & -1 & -5 \\ 6 & 7 & 9 \end{bmatrix}$$

(i) 364

(ii) 14

(iii) 76

(iv) 340

(f) A Hamiltonian cycle in a Hamiltonian graph of order 24 has :

(i) 12 edges

(ii) 24 edges

(iii) 23 edges

(iv) None of the above

(g) The size of a forest, F, with 17 vertices and 4 components is :

(i) 4

(ii) 16

(iii) 14

(iv) None of the above

(h) The number of spanning trees in the complete graph K_2 :

(i) 48

(ii) 8^6

(iii) 6^8

(iv) None of the above

(i) Find the value of $2a$ if the following matrix is singular :

$$\begin{bmatrix} -4 & 2 \\ -6 & \alpha \end{bmatrix}$$

(i) $1/3$

(ii) $-1/3$

(iii) 0

(iv) 3

Group – B

(Long-answer Type Questions)

Answer any **four** questions : $15 \times 4 = 60$

2. Let A, B and C be sets. Then show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
3. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.
4. In a group of 50 students, the number of students studying French, English and Sanskrit were found to be as follows :
French = 17, English = 13, Sanskrit = 15, French and English = 09, English and Sanskrit = 4, French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study :

(a) French only

- (b) English only
 - (c) Sanskrit only
 - (d) English and Sanskrit
 - (e) French and Sanskrit but not English
 - (f) French and English but not Sanskrit
 - (g) At least one of the three languages
 - (h) None of the three languages but not French
5. (a) State and prove both De Morgan's Laws for two sets.
6. Prove the following using truth-tables :
- (a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
 - (b) $\sim (p \vee q) \equiv \sim p \wedge \sim q$
7. Prove that inverse of product of the two matrices (if conformal for product) is equal to product of their inverses taken in reverse order.
8. Explain about the following :
- (a) Tautology
 - (b) Contradiction

- (c) Equivalence
- (d) Implication
- (e) Negation

9. Define, with example, the following :

- (a) Null Graph
- (b) Complete Graph
- (c) Complete-Bipartite Graph
- (d) Path
- (e) Cycle

10. Solve the following system of equations by matrix inversion method :

- (a) $x + y + 2z = 4$
- (b) $2x - y + 3z = 9$
- (c) $3x - y - z = 2$

s
yf



d.